

M8 Übungsblatt zur 1. Schulaufgabe: Algebra – Lösungsvorschlag (Ohne Gewähr!) Bru

$$\begin{aligned} 0.1) \quad -7x - [8 + 12x - 27] &= 50 - 18x - 6[-2x + 3] \\ -7x + 19 - 12x &= 50 - 18x + 12x - 18 \\ -19x + 19 &= 32 - 6x \quad / + 19x - 32 \\ 13x &= -13 \quad / : 13 \\ x &= -1 \end{aligned}$$

$$\begin{aligned} 0.2) \quad 4x - 3 + 27 - 54x &= 10 - 30x \\ -50x + 24 &= 10 - 30x \quad / + 50x - 10 \\ 14 &= 20x \quad / : 20 \\ x &= 0,7 \end{aligned}$$

$$0.3) \frac{9}{16}p^2 - \frac{6}{5}pq + \frac{16}{25}q^2 \qquad 0.4) 20,25c^2 - 28,09d^2$$

$$0.5) \dots = 16x^2 - 40x + 25 - 36x^2 - 84x - 49 + 20x^2 - 80 = -124x - 104$$

$$\begin{aligned} 0.6) \quad \dots &= \frac{1}{2}(16x^2 - 12xy + 2,25y^2) - \frac{3}{4}(-1)\left(4y^2 - 2xy + \frac{1}{4}x^2\right) - \frac{9}{4}x^2 + 9xy - 9y^2 = \\ &= 8x^2 - 6xy + \frac{9}{8}y^2 + 3y^2 - \frac{3}{2}xy + \frac{3}{16}x^2 - \frac{9}{4}x^2 + 9xy - 9y^2 = \\ &= 5\frac{15}{16}x^2 + \frac{3}{2}xy - 4\frac{7}{8}y^2 \end{aligned}$$

$$\begin{aligned} 0.7) \quad 4x^2 + 44x + 121 - 49x^2 + 126x - 81 &= 196 - 9x^2 - 36x^2 + 192x - 256 \\ -45x^2 + 170x + 40 &= -45x^2 + 192x - 60 \quad / + 45x^2 - 170x + 60 \\ 100 &= 22x \quad / : 22 \\ x &= 4\frac{6}{11} \end{aligned}$$

$$0.8) 3ax(2x^2 - 3x + 4) \quad 0.9) \frac{5}{2}st\left(1 - \frac{5}{3}st + \frac{3}{2}s^2\right) \quad 0.10) 14a^2b^2(14 - 8a + 3b)$$

$$0.11) (m - n)(a - c) \quad 0.12) n(2s - t) - p(2s - t) = (2s - t)(n - p)$$

$$0.13) 4c^2(x - 3y) - 3d^2(x - 3y) = (x - 3y)(4c^2 - 3d^2)$$

$$\begin{aligned} 0.14) \quad (2a - 4b + 7c)(7u - 3v + 2w) + [(4a + 6b - 9c)(-4)](7u - 3v + 2w) &= \\ = (7u - 3v + 2w)[(2a - 4b + 7c) + (-4)(4a + 6b - 9c)] &= \\ = (7u - 3v + 2w)[2a - 4b + 7c - 16a - 24b + 36c] &= \\ = (7u - 3v + 2w)(-14a - 28b + 43c) & \end{aligned}$$

$$0.15) (a - 4)^2 \quad 0.16) (a^5 + 9)^2 \quad 0.17) (1,4k^2 - 1)^2 \quad 0.18) \left(\frac{5}{6}m - \frac{7}{9}n\right)\left(\frac{5}{6}m + \frac{7}{9}n\right)$$

$$0.19) 5(4a^2 - 25b^2) = 5(2a - 5b)(2a + 5b)$$

$$0.20) 4k(k^2 + 6k + 9) = 4k(k + 3)^2$$

$$0.21) 14\left(y^2 + y + \frac{1}{4}\right) = 14\left(y + \frac{1}{2}\right)^2$$

$$0.22) \frac{1}{2}\left(9z^2 - 20z + \frac{100}{9}\right) = \frac{1}{2}\left(3z + \frac{10}{3}\right)^2$$

$$0.23) (x^2 - y^2)(x^2 + y^2) = (x - y)(x + y)(x^2 + y^2) \quad 0.24) (4c^2 - 9d^2)^2 = (2c - 3d)^2(2c + 3d)^2$$

$$0.25) (x + y)^2 - z^2 = [(x + y) - z][(x + y) + z] = (x + y - z)(x + y + z)$$

$$1.1) D = \mathbb{Q} \setminus \{3\} \quad 1.2) 4x^2 - 7x = x(4x - 7) = 0 \text{ für } x = 0 \text{ oder } x = \frac{7}{4} \Rightarrow D = \mathbb{Q} \setminus \left\{0; \frac{7}{4}\right\}$$

$$1.3) x^2 - 6x + 9 = (x - 3)^2 = 0 \text{ für } x = 3 \Rightarrow D = \mathbb{Q} \setminus \{3\} \quad 1.4) \frac{b^3}{13a^2c^2}$$

$$1.5) \frac{7yz}{7yz(2yz^2 - 3y^2z)} = \frac{1}{2yz^2 - 3y^2z} \quad 1.6) \frac{3(1-2x)}{-9(1-2x)} = -\frac{1}{3} \quad 1.7) \frac{14c(4c-5)}{3x(4c-5)} = \frac{14c}{3x}$$

$$1.8) \frac{2a^2(1-2ab+3a^2b)}{2a^2(9ab-4+3b^2)} = \frac{1-2ab+3a^2b}{9ab-4+3b^2} \quad 1.9) \frac{(x-3)(x+3)}{5y(x-3)} = \frac{x+3}{5y}$$

$$1.10) \frac{a(a-2)(a+2)}{3(2-a)} = -\frac{a(a+2)}{3} \quad 1.11) \frac{(a-b)^2}{(a^2+b^2)(a-b)(a+b)} = \frac{(a-b)}{(a^2+b^2)(a+b)}$$

$$1.12) \frac{(8x-6y)(8x+6y)}{(8x+6y)^2} = \frac{8x-6y}{8x+6y} = \frac{4x-3y}{4x+3y} \quad 1.13) \frac{x(x-6)^2}{x^2(6-x)(6+x)} = -\frac{x(x-6)}{x^2(x+6)} = -\frac{x-6}{x(x+6)}$$

$$1.14) \frac{(1-c^2)(1+c^2)}{c^2(c+1)+1(c+1)} = \frac{(1-c)(1+c)(1+c^2)}{(1+c^2)(1+c)} = 1-c$$

$$1.15) \frac{a^2-4}{a(a-2)} \quad 1.16) \frac{4a^3+7a^2b-2ab^2}{a(a+b)(a+2b)} \quad 1.17) \frac{6a^3b^2c^2}{240a^2b^2c} \quad 1.18) \frac{55ab^2+85b^3}{240a^2b^2c}$$

$$1.19) \frac{30x^2-150x}{20x(x+5)(x-5)} \text{ erweitert mit } 10x(x-5) \quad 1.20) \frac{50x^2+250x}{20x(x+5)(x-5)} \text{ erweitert mit } 50(x+5)$$

$$1.21) \frac{4x^3y^5}{36x^2y^3z^3}, \frac{6y^4z^5}{36x^2y^3z^3}, \frac{3x^4z}{36x^2y^3z^3}$$

| | | |
|-------|---|--|
| 1.22) | Zerlegung $2c - 4 = 2(c - 2)$ $6 - 3c = 3(2 - c)$ $c^2 - 4 = (c - 2)(c + 2)$ | Erweiterungsfaktoren $3(c + 2)$ $(-2)(c + 2)$ 6 |
|-------|---|--|

$$\text{HN} = 6(c - 2)(c + 2)$$

$$\text{Ergebnis: } \frac{3c^2 + 9c + 6}{6(c - 2)(c + 2)}, \frac{-2c^2 - 2c + 4}{6(c - 2)(c + 2)}, \frac{12}{6(c - 2)(c + 2)}$$

$$1.23) \text{HN} = (4 - x)(4 + x) \quad \frac{x^2 + 8x + 16}{(4 - x)(4 + x)}, \frac{-x^2 + 8x - 16}{(4 - x)(4 + x)}, \frac{8}{16 - x^2}$$

$$1.24) \text{HN} = (y - 1)^2(y + 1) \quad \frac{y^2 + y}{(y - 1)^2(y + 1)}, \frac{5y - 5}{(y - 1)^2(y + 1)}, \frac{2y^2 - 4y + 2}{(y - 1)^2(y + 1)}$$

1.25)

| | |
|---|--|
| Zerlegung $a^2 - ab = a(a - b)$ $5b^2 - 5ab = 5b(b - a)$ $a^3b^2 - a^2b^3 = a^2b^2(a - b)$ | Erweiterungsfaktoren $5ab^2$ $(-1)a^2b$ 5 |
|---|--|

$$\text{HN} = 5a^2b^2(a - b)$$

$$\text{Ergebnisse: } \frac{30ab^2}{5a^2b^2(a - b)}, \frac{-3a^3b}{5a^2b^2(a - b)}, \frac{15}{5a^2b^2(a - b)}$$

$$2.1) \dots = \frac{3a-7b}{a-2b} + \frac{3b-a}{a-2b} = \frac{2a-4b}{a-2b} = 2$$

$$2.2) \dots = \frac{(u+3v)^2}{(u-3v)^2} - \frac{(u-3v)^2}{(u-3v)^2} = \frac{u^2+6uv+9v^2-u^2+6uv-9v^2}{(u-3v)^2} = \frac{12uv}{(u-3v)^2}$$

$$2.3) \text{HN} = 72a^2c^2 \quad \dots = \frac{(3+4a) \cdot 9ac}{\text{HN}} + \frac{(4ac-9a) \cdot 3c}{\text{HN}} - \frac{(5c^2-9ac) \cdot 4a}{\text{HN}} =$$

$$= \frac{27ac + 36a^2c + 12ac^2 - 27ac - 20ac^2 + 36a^2c}{\text{HN}} = \frac{72a^2c - 8ac^2}{\text{HN}} =$$

$$= \frac{8ac(9a-c)}{72a^2c^2} = \frac{9a-c}{9ac}$$

$$2.4) \text{HN} = 6a(a-3)$$

$$\dots = \frac{3a(6-a) + (9a^2-30a) - 6a(a-3)}{\text{HN}} = \frac{18a-3a^2+9a^2-30a-6a^2+18a}{\text{HN}} =$$

$$= \frac{6a}{6a(a-3)} = \frac{1}{a-3}$$

$$2.5) \dots = \frac{(x-y)(5x-3) - (y+x)(5x+3) + (4y+6)x}{(5x-3)(5x+3)} =$$

$$= \frac{5x^2-3x-5xy+3y-5xy-3y-5x^2-3x+4xy+6x}{\text{HN}} = \frac{6xy}{(5x-3)(5x+3)}$$

2.6)

Bestimmung des Hauptnenners:

EWF:

$$6a+9b = 3(2a+3b) \quad 10(2a-3b)$$

$$20a-30b = 10(2a-3b) \quad 3(2a+3b)$$

$$\underline{8a^2-18b^2 = 2(2a-3b)(2a+3b) \quad 15}$$

$$\text{HN} = 30(2a-3b)(2a+3b)$$

$$\dots = \frac{(6a+8b)(20a-30b) - (5a-2b)(6a+9b) - 15(6a^2-3ab-14b^2)}{\text{HN}} =$$

$$= \frac{120a^2-20ab-240b^2-30a^2-33ab+18b^2-90a^2+45ab+210b^2}{\text{HN}} =$$

$$= \frac{8ab-12b^2}{30(2a-3b)(2a+3b)} = \frac{4b(2a-3b)}{30(2a-3b)(2a+3b)} = \frac{b}{15(2a+3b)}$$

$$2.7) \text{HN} = 2(4a-5)(4a+5)$$

$$\dots = \frac{2(4a-5)^2 + (-2)(4a+5)^2 + 44a+145}{\text{HN}} =$$

$$= \frac{32a^2-80a+50-32a^2-80a-50+44a+145}{\text{HN}} =$$

$$= \frac{-116a+145}{2(4a-5)(4a+5)} = \frac{(-29)(4a-5)}{2(4a-5)(4a+5)} = \frac{-29}{2(4a+5)}$$

$$2.8) \text{HN} = 7x(x-3)(x+3) \quad \dots = \frac{9+6x^2-7x^2+x^2-9}{\text{HN}} = \frac{0}{\text{HN}} = 0$$

2.9)

Hauptnenner:

Erweiterungsfaktoren:

$$x^4-1 = (x-1)(x+1)(x^2+1)$$

$$1$$

$$x^2-1 = (x-1)(x+1)$$

$$(x^2+1)$$

$$x-1$$

$$(x+1)(x^2+1)$$

$$\text{HN} = (x-1)(x+1)(x^2+1)$$

$$\begin{aligned} \dots &= \frac{x^4 + 1 + (x^2 + 1)^2 - (x + 1)^2(x^2 + 1)}{\text{HN}} = \\ \dots &= \frac{x^4 + 1 + x^4 + 2x^2 + 1 - (x^2 + 2x + 1)(x^2 + 1)}{\text{HN}} = \\ \dots &= \frac{x^4 + 1 + x^4 + 2x^2 + 1 - x^4 - x^2 - 2x^3 - 2x - x^2 - 1}{\text{HN}} = \\ &= \frac{x^4 - 2x^3 - 2x + 1}{\text{HN}} \end{aligned}$$

2.10)

Hauptnenner:

$$3a + 9 = 3(a + 3)$$

$$6a^2 - 54 = 6(a + 3)(a - 3)$$

$$9 - a^2 = (3 - a)(3 + a) = (-1)(a - 3)(a + 3) \quad \underline{\underline{(-6)}}$$

$$\text{HN} = 6(a + 3)(a - 3)$$

$$\begin{aligned} \dots &= \frac{2(a - 3)(3 - 2a) - (11a^2 - 9a + 54) - (-6)(12 - 12a)}{\text{HN}} = \\ &= \frac{6a - 4a^2 - 18 + 12a - 11a^2 + 9a - 54 + 72 - 72a}{\text{HN}} = \frac{-45a - 15a^2}{\text{HN}} = \frac{(-15a)(3 + a)}{6(a + 3)(a - 3)} = \\ &= -\frac{5a}{2(a - 3)} \end{aligned}$$

2.11)

Hauptnenner:

$$3x - 4y$$

$$24xy - 9x^2 - 16y^2 = (-1)(3x - 4y)^2$$

$$9x^2 + 24xy + 16y^2 = (3x + 4y)^2 \quad \underline{\underline{(3x - 4y)^2}}$$

$$\text{HN} = (3x - 4y)^2(3x + 4y)^2$$

$$\begin{aligned} &\frac{1}{3x - 4y} - \frac{y - 2x}{24xy - 9x^2 - 16y^2} + \frac{3y - x}{9x^2 + 24xy + 16y^2} = \\ &= \frac{(9x^2 - 16y^2)(3x + 4y) - (-1)(y - 2x)(9x^2 + 24xy + 16y^2) + (3y - x)(9x^2 - 24xy + 16y^2)}{\text{HN}} \\ &= \frac{27x^3 + 36x^2y - 48xy^2 - 64y^3 + 9x^2y + 24xy^2 + 16y^3 - 18x^3 - 48x^2y - 32xy^2 +}{\text{HN}} \\ &\quad + 27x^2y - 72xy^2 + 48y^3 - 9x^3 + 24x^2y - 16xy^2 = \\ &= \frac{48x^2y - 144xy^2}{\text{HN}} = \frac{48xy(x - 3y)}{(3x - 4y)^2(3x + 4y)^2} \end{aligned}$$

Erweiterungsfaktoren:

$$2(a - 3)$$

$$1$$

$$(-6)$$

Erweiterungsfaktoren:

$$(3x - 4y)(3x + 4y)^2$$

$$(-1)(3x + 4y)^2$$

$$(3x - 4y)^2$$

$$3.1) \frac{10x^3}{9a^2} \quad 3.2) -\frac{3c}{2} \quad 3.3) \dots = \frac{(7a-1) \cdot 3(a+3)}{(a-3)(a+3) \cdot 4a(7a-1)} = \frac{3}{4a(a-3)}$$

$$3.4) \dots = \left(\frac{4d^2 - c^2}{4c^2d^2} \right) \cdot \frac{c+2d}{c-2d} = \frac{(2d-c)(2d+c)(c+2d)}{4c^2d^2(c-2d)} = -\frac{(c+2d)^2}{4c^2d^2}$$

$$3.5) \dots = \frac{4}{9x^2} - \frac{3}{xy} + \frac{81}{16y^2} = \frac{64y^2 - 432xy + 729y^2}{144x^2y^2}$$

$$3.6) \dots = \frac{a^2 - 2ab + 2ab + 4b^2}{(a-2b)(a+2b)} \cdot \frac{a^2 - 4b^2}{2ab} = \frac{a^2 + 4b^2}{2ab}$$

$$3.7) \dots = \frac{3(a+4d)}{5(y-3)(y+3)} \cdot \frac{10(y-3)^2}{4a(a+4d)} = \frac{3(y-3)}{2a(y+3)} \quad 3.8) \dots = \frac{y+x}{xy} : \frac{y}{x} = \frac{y+x}{xy} \cdot \frac{x}{y} = \frac{y+x}{y^2}$$

$$3.9) \dots = \left(\frac{u^2 - v^2}{u} \right) \cdot \frac{2uv}{v-u} = \frac{(u-v)(u+v)}{u} \cdot \frac{2uv}{(-1)(u-v)} = -2v(u+v)$$

$$3.10) \dots = \left(\frac{xy(y-x) - y^2(y+x)}{y(y+x)(y-x)} \right) \cdot \frac{4(y-x)^2}{3(x^2+y^2)} = \frac{xy^2 - x^2y - y^3 - xy^2}{y(y+x)(y-x)} \cdot \frac{4(y-x)^2}{3(x^2+y^2)} =$$

$$= \frac{(-y)(x^2+y^2)}{y(y+x)(y-x)} \cdot \frac{4(y-x)^2}{3(x^2+y^2)} = -\frac{4(y-x)}{3(y+x)} = \frac{4(x-y)}{3(x+y)}$$

$$3.11) \dots = \frac{b(a-b)}{(a-b)^2} \cdot \frac{1}{a-b} - \frac{b-a}{(b-a)(b+a)} \cdot \frac{a+b}{a} = \frac{b}{(a-b)^2} - \frac{1}{a} =$$

$$= \frac{ab - a^2 - b^2 + 2ab}{a(a-b)^2} = \frac{3ab - a^2 - b^2}{a(a-b)^2}$$

$$3.12) \dots = \left(\frac{x^2 - y^2}{xy} \right) : \left(\frac{x-y}{xy} \right) = \frac{(x-y)(x+y)}{xy} \cdot \frac{xy}{x-y} = x+y$$

4. Bruchgleichungen

$$D = \mathbb{Q} \setminus \{0\} \quad \text{HN} = 45x$$

$$\frac{27-15x}{45x} = \frac{30-20x}{45x} \quad | \cdot 45x$$

$$27 - 15x = 30 - 20x$$

$$5x = 3$$

$$x = \frac{3}{5} ; \quad L = \left\{ \frac{3}{5} \right\}$$